SU(3) Breaking Effects in Charmed Meson Decays

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Abstract

The decay rates of $D^+ \to \pi^+\pi^0$ and $D^0 \to K^+\pi^-$ recently measured by CLEO give the ratios $R_1 = 2|V_{cs}/V_{cd}|^2\Gamma(D^+ \to \pi^+\pi^0)/\Gamma(D^+ \to \bar{K}^0\pi^+) = 3.29 \pm 1.16$ and $R_2 = |V_{cs}^*V_{ud}/(V_{cd}^*V_{us})|^2\Gamma(D^0 \to K^+\pi^-)/\Gamma(D^0 \to K^-\pi^+) = 2.92 \pm 1.34$. Both are about three times of those expected from SU(3) symmetry. We show that, in the large-N_c factorization approach, such large SU(3) violations can be accounted for by the accumulations of several small SU(3)-breaking effects. An important requirement is the relative magnitude of the form factors, $f_+^{D\pi}(0) > f_+^{DK}(0)$.

For a long time, theorists [1-3] had observed that, if SU(3) is a good symmetry, the ratios

$$R_1 = 2 \left| \frac{V_{cs}}{V_{cd}} \right|^2 \frac{\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to \bar{K}^0\pi^+)} \tag{1}$$

and

$$R_2 = \left| \frac{V_{cs}^* V_{ud}}{V_{cd}^* V_{us}} \right|^2 \frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} \tag{2}$$

both ought to be unity. Especially R_1 being unity, as had been emphasized all along, is a clean SU(3) prediction, since both channels are exotic and there should be no final-state-interaction effects to alter the SU(3) results. Quite strikingly, recent data of CLEO

$$Br(D^+ \to \pi^+\pi^0) = (0.22 \pm 0.05 \pm 0.05)\% \quad [4], \tag{3}$$

$$Br(D^0 \to K^+\pi^-) = (0.77 \pm 0.25 \pm 0.25)\% \times Br(D^0 \to K^-\pi^+) \quad [5], \tag{4}$$

give the values of these ratios

$$R_1 = 3.29 \pm 1.16, \tag{5}$$

$$R_2 = 2.92 \pm 0.95 \pm 0.95, \tag{6}$$

far exeeding unity; a clear large violation of SU(3) results. [In obtaining Eqs.(5) and (6), we have used the branching ratio $Br(D^+ \to \bar{K}^0\pi^+) = (2.6 \pm 0.4)\%$ given by the Particle Data Group (PDG) [6] and $(V_{cd}/V_{cs})^2 = 0.05138$.]

We show in this paper that, the larger-than-unity values of R_1 and R_2 can be accounted for in the the large N_c factorization approach; the net large SU(3)-symmetry violations in R_1 and R_2 are the cumulative results of several small SU(3)-breaking effects [7].

Let us consider first the decay modes $D^+ \to \pi^+\pi^0$ and $D^+ \to \bar{K}^0\pi^+$ in terms of the quark-diagram amplitudes [2,3],

$$A(D^+ \to \pi^+\pi^0) = -\frac{G_F}{2} V_{cd}^* V_{ud} (\mathcal{A} + \mathcal{B})_{\pi\bar{\pi}} e^{i\delta_2^{\pi\pi}}, \tag{7}$$

$$A(D^+ \to \bar{K}^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\mathcal{A} + \mathcal{B})_{\bar{K}\pi} e^{i\delta_{3/2}^{\bar{K}\pi}}, \tag{8}$$

where $\mathcal{A}$ is the external W-emission amplitude and $\mathcal{B}$ is the internal W-emission amplitude. These quark-diagram amplitudes have well-defined meaning with all QCD strong-interaction effects included. The final-state interactions are expressed by the phase shifts, the δ's,

which in general have both real and imaginary parts; the real parts are related to the elastic scattering effects while the imaginary parts indicate effects of inelasticity [4]. Since both $\pi^+\pi^0$ and $\bar{K}^0\pi^+$ channels are exotic, we expect that the phase shifts $\delta_2^{\pi\bar{\pi}}$ and $\delta_{3/2}^{\bar{K}\pi}$ in Eqs.(7) and (8) are negligible (actually all we have used in this analysis is that they are real). If SU(3) symmetry is good, i.e. $(\mathcal{A}+\mathcal{B})_{\pi\bar{\pi}} = (\mathcal{A}+\mathcal{B})_{\bar{K}\pi}$, it is easily seen that $R_1 = |\mathcal{A}+\mathcal{B}|^2_{\pi\bar{\pi}} \big/ |\mathcal{A}+\mathcal{B}|^2_{\bar{K}\pi} = 1$ by substituting Eqs.(7) and (8) into Eq.(1). That was the result given in Refs.[1,2] more than a decade ago, (in Ref.[1] the invariant SU(3) amplitudes were used; in Ref.[2] the quark diagram scheme was introduced and used as presented here.)

We can determine the absolute values of the quark diagram amplitude $|\mathcal{A}+\mathcal{B}|$ in Eqs.(7) and (8) from the aforementioned branching ratios for $D^+ \to \pi^+\pi^0$ and $D^+ \to \bar{K}^0\pi^+$:

$$|\mathcal{A}+\mathcal{B}|_{\pi\bar{\pi}} = (0.287 \pm 0.065)\,\mathrm{GeV}^3, \tag{9}$$

$$|\mathcal{A}+\mathcal{B}|_{\bar{K}\pi} = (0.164 \pm 0.018)\,\mathrm{GeV}^3, \tag{10}$$

where we have used the total width $\Gamma(D^+) = 6.174 \times 10^{-13}$ GeV [6] to covert the branching ratios into decay rates. Clearly they are far from being equal, thus violating SU(3) symmetry.

In the following we try to understand this large SU(3)-symmetry violation in these quark-diagram amplitudes from the state-of-the-art large N_c factorization approach (for a review, see [8]). The quark-diagram amplitudes are given by

$$
\begin{aligned}
\mathcal{A}_{\pi\bar{\pi}} &= -\sqrt{2}a_1(p_{\pi^+})_\mu f_\pi \langle \pi^0 | \bar{c}\gamma^\mu(1-\gamma_5)d | D^+ \rangle, \\
\mathcal{B}_{\pi\bar{\pi}} &= a_2(p_{\pi^0})_\mu f_\pi \langle \pi^+ | \bar{c}\gamma^\mu(1-\gamma_5)u | D^+ \rangle, \\
\mathcal{A}_{\bar{K}\pi} &= a_1(p_{\pi^+})_\mu f_\pi \langle \bar{K}^0 | \bar{c}\gamma^\mu(1-\gamma_5)s | D^+ \rangle, \\
\mathcal{B}_{\bar{K}\pi} &= a_2(p_{\bar{K}^0})_\mu f_\pi \langle \pi^+ | \bar{c}\gamma^\mu(1-\gamma_5)u | D^+ \rangle,
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
a_1 &= \frac{1}{2}(c_+ + c_-) + \frac{1}{2N_c}(c_+ - c_-), \\
a_2 &= \frac{1}{2}(c_+ - c_-) + \frac{1}{2N_c}(c_+ + c_-),
\end{aligned}
\tag{12}
$$

c_+ and c_- are the Wilson coefficient functions to be evaluated at the renormalization scale $\mu \sim m_c$, and N_c is the number of quark color degrees of freedom. The two-body matrix elements in Eq.(11) are in general expressed in terms of the form factors f_0 and f_1 [9]; for

example,

$$\langle \pi^0 | \bar{c}\gamma_\mu d | D^+ \rangle = -\frac{1}{\sqrt{2}} \left\{ \left((p_D + p_\pi)_\mu - \frac{m_D^2 - m_\pi^2}{q^2} q_\mu \right) f_1^{D\pi}(q^2) + \frac{m_D^2 - m_\pi^2}{q^2} q_\mu f_0^{D\pi}(q^2) \right\}, \quad (13)$$

where $q = p_D - p_\pi$ and the factor of $-\frac{1}{\sqrt{2}}$ comes from the wave function of the π^0. Substituting Eqs.(11) and (13) into Eqs.(7) and (8) we obtain

$$
\begin{aligned}
(\mathcal{A} + \mathcal{B})_{\pi\bar{\pi}} &= (m_D^2 - m_\pi^2) f_\pi f_0^{D\pi}(m_\pi^2)(a_1 + a_2), \\
(\mathcal{A} + \mathcal{B})_{\bar{K}\pi} &= a_1(m_D^2 - m_K^2) f_\pi f_0^{DK}(m_\pi^2) + a_2(m_D^2 - m_\pi^2) f_K f_0^{D\pi}(m_K^2).
\end{aligned}
\quad (14)
$$

As noted before, it is reasonable to assume that $\delta_2^{\pi\bar{\pi}} \sim 0$ and $\delta_{3/2}^{\bar{K}\pi} \sim 0$ since these are exotic channels. Hence, we find from Eqs.(1), (7), (8) and (14) that

$$R_1 = 1.073 \left(\frac{[m_D^2 - m_\pi^2]}{[m_D^2 - m_K^2]} \frac{f_0^{D\pi}(m_\pi^2)}{f_0^{DK}(m_\pi^2)} \frac{[1 + (a_2/a_1)]}{[1 + (a_2/a_1)r]} \right)^2, \quad (15)$$

where the factor of 1.073 comes from the phase-space differences for the $\pi\bar{\pi}$ and $\bar{K}\pi$ modes, and

$$r = \frac{f_K}{f_\pi} \frac{m_D^2 - m_\pi^2}{m_D^2 - m_K^2} \frac{f_0^{D\pi}(m_K^2)}{f_0^{DK}(m_\pi^2)}. \quad (16)$$

In order to calculate $(\mathcal{A} + \mathcal{B})$ and R_1, we need information on the form factors and c_1, c_2. The q^2 dependence of the form factor $f_0(q^2)$ is usually assumed to be governed by a single low-lying pole:

$$f_0(q^2) = \frac{f_0(0)}{1 - (q^2/m_*^2)}, \quad (17)$$

where m_* is the mass of the 0^+ pole. For the form factor $f_0^{DK}(0)$ at $q^2 = 0$, we use the average value (recall that $f_0(0) = f_1(0) = f_+(0)$; see [9])

$$f_0^{DK}(0) = 0.76 \pm 0.02 \quad (18)$$

extracted from the recent measurements of $D \to K\ell\bar{\nu}$ by CLEO II, E687 and E691 [10]. For the form factor $f_0^{D\pi}(0)$, there are various sources of information: A previous measurement of the Cabibbo-suppressed decay $D^0 \to \pi^- \ell^+ \nu$ by Mark III [11,10] yields

$$\left| \frac{f_0^{D\pi}(0)}{f_0^{DK}(0)} \right| = 1.0^{+0.6}_{-0.3} \pm 0.1 \, ; \quad (19)$$

4

a very recent CLEO-II measurement of $D^+ \to \pi^0 \ell^+ \nu$ [12] gives

$$\left| \frac{f_0^{D\pi}(0)}{f_0^{DK}(0)} \right| = 1.29 \pm 0.21 \pm 0.11 \,, \tag{20}$$

which clearly indicates that $f_0^{D\pi}(0)$ is most likely greater than $f_0^{DK}(0)$, in agreement with the current theoretically favored value $f_0^{D\pi}(0)/f_0^{DK}(0) = 1.18$ given by the heavy quark symmetry and chiral perturbation theory [13] and ruling out the previous Wirbel-Stech-Bauer results $f_0^{D\pi}(0) = 0.69$ and $f_0^{DK}(0) = 0.76$ [14]. For definiteness, we use $f_0^{D\pi}(0) = 0.83$ determined from the experimental result Eq.(3) for $D^+ \to \pi^+ \pi^0$. Moreover, we use $f_\pi = 132$ MeV and $f_K = 161$ MeV [6].

It is known from analyzing the data of charm decays using the quark diagram scheme [3,15] that charm decays involving the internal W-emission diagram are not suppressed, though some earlier model-calculations had such suppressions [16] and gave birth to the so-called "color-suppression" rule. The cleanest example of such decay is $D^0 \to \bar{K}^0 \phi$ which involves only the internal W-emission diagram. The so-called QCD-sum-rule calculation of Blok and Shifman [16] gave too small branching ratio for it. As far as we know, such defect in their QCD-sum-rule calculations has not being corrected for charmed meson decays. Future measurement of $D^+ \to K^+ \phi$, another decay involving only the internal W-emission diagram, will further clarify this point. (See Ref.[15] for detailed discussions on this point and critics on Blok and Shifman's QCD-sum-rule calculations for charmed meson decays).

The absence of suppression on the internal W-emission diagrams in charmed meson decays, together with the destructive interference in the decay of $D^+ \to \bar{K}^0 \pi^+$, was later identified in the large-N_c factorization model calculation [17] to the dropping of the $1/N_c$ terms (for a review, see [8]). Taught by these experiences, we will neglect the $1/N_c$ correction in our calculations here.

Given $c_+(m_c) \cong 0.75$, $c_-(m_c) \cong 1.77$ [18], and neglecting the $1/N_c$ term, we obtain from Eq.(12)

$$a_1(m_c) = \frac{1}{2}(c_+ + c_-) \cong 1.26 \,, \quad a_2(m_c) = \frac{1}{2}(c_+ - c_-) \cong -0.51 \,. \tag{21}$$

Using $m_* = 2.47$ GeV for $D \to \pi$ transition and $m_* = 2.60$ GeV for $D \to K$, [14], we finally arrive at

$$R_1 = 3.3 \,, \tag{22}$$

in excellent agreement with the experimental result Eq.(5).

The ratio R_1 is very sensitive to the relative magnitude of the form factors $f_0^{D\pi}(0)$ and $f_0^{DK}(0)$. For example, if the previous Wirbel-Stech-Bauer results $f_0^{D\pi}(0) = 0.69$ and $f_0^{DK}(0) = 0.76$ [14] were used in our calculation, we would have obtained $R_1 = 1.4$, in disagreement with data. So the value of the form factor ratio needed in our analysis is consistent with the experimental value of Eq.(20). We further note in Eq.(15) that the ratio of $\mathcal{B}/\mathcal{A}$ is given by $a_2/a_1 \cong -0.40$ for $D^+ \to \pi^+\pi^0$ decay and by $(a_2/a_1)r \cong -0.60$ for $D^+ \to \bar{K}^0\pi^+$ decay; i.e. the destructive interference in the latter decay is much more severe than in the former, a SU(3)-symmetry violation effect crucial to making the value of R_1 much greater than unity.

To summerize, we see from Eqs.(15) and (16) that all the SU(3)-symmetry breaking effects, though small individually, enhance cumulatively the decay rate of $D^+ \to \pi^+\pi^0$ in comparsion to that of $D^+ \to \bar{K}^0\pi^+$: $(m_D^2 - m_\pi^2)/(m_D^2 - m_K^2) = 1.07$, $f_0^{D\pi}(m_\pi^2)/f_0^{DK}(m_\pi^2) = 1.092$, $f_0^{D\pi}(m_K^2)/f_0^{DK}(m_\pi^2) = 1.134$, $f_K/f_\pi = 1.22$; each of them gives a SU(3)-symmetry effect below 25%, but the accumulation of them leads to $[1 + (c_2/c_1)]/[1 + (c_2/c_1)r] = 1.5$ and finally a value of $R_1 = 3.3$.

We next discuss the decays $D^0 \to K^+\pi^-$ and $D^0 \to K^-\pi^+$. In terms of the quark diagram amplitudes [2,3]

$$A(D^0 \to K^-\pi^+) = \frac{1}{3}\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[(2\mathcal{A} - \mathcal{B} + 3\mathcal{C})_{\bar{K}\pi}e^{i\delta_{1/2}^{\bar{K}\pi}} + (\mathcal{A} + \mathcal{B})_{\bar{K}\pi}e^{i\delta_{3/2}^{\bar{K}\pi}}\right], \qquad (23)$$

where $\mathcal{C}$ is the W-exchange amplitude. The expression for $A(D^0 \to K^+\pi^-)$ is exactly the same as Eq.(23) except replacing $V_{cs}^*V_{ud}$ by $V_{cd}^*V_{us}$ and the subscript and superscript $\bar{K}\pi$ by $K\bar{\pi}$. It is obvious that if the square brackets in Eq.(23) for $\bar{K}\pi$ and $K\bar{\pi}$ are the same, then $R_2 = 1$.

To calculate R_2 we need to have know the phase shift difference $\Delta_{\bar{K}\pi} = \delta_{1/2}^{\bar{K}\pi} - \delta_{3/2}^{\bar{K}\pi}$, i.e. the final-state-interaction effects. For the $\bar{K}\pi$ channel both the the phase shift difference $\Delta_{\bar{K}\pi}$ and the quark-diagram amplitudes $|\mathcal{A} + \mathcal{B}|$, $|\mathcal{B} - \mathcal{C}|$ in Eq.(23) can be obtained from the available data of $D^+ \to \bar{K}^0\pi^+$, $D^0 \to K^-\pi^+$ and $\bar{K}^0\pi^0$. Using the PDG values for the decay rates of $D^+ \to \bar{K}^0\pi^+$, $D^0 \to \bar{K}^0\pi^0$ [6] and the updated result for the branching ratio of $D^0 \to K^-\pi^+$ [19], $Br(D^0 \to K^-\pi^+) = (3.90 \pm 0.16)\%$, which is the weighted average of previous measurements and the new CLEO result of $(3.95 \pm 0.08 \pm 0.17)\%$ [19], we obtain

(for simplicity, $\delta_{1/2}^{\bar{K}\pi}$ and $\delta_{3/2}^{\bar{K}\pi}$ are assumed to be real)

$$|\mathcal{B} - \mathcal{C}|_{\bar{K}\pi} = (0.205 \pm 0.010)\,\mathrm{GeV}^3, \quad \Delta_{\bar{K}\pi} = (90 \pm 11)^\circ, \tag{24}$$

with $|\mathcal{A} + \mathcal{B}|_{\bar{K}\pi}$ being given by Eq.(10). Unfortunately, we cannot do a similar detailed quark-diagram analysis for $D^0 \to K^+\pi^-$ to obtain $|\mathcal{B} - \mathcal{C}|_{K\bar{\pi}}$ and the phase shift difference $\Delta_{K\bar{\pi}} = \delta_{1/2}^{K\bar{\pi}} - \delta_{3/2}^{K\bar{\pi}}$ directly from experiment in comparison with those of $\bar{K}\pi$, since other quark-mixing doubly-suppressed decays have not been measured. Since we do not know how to calculate phase shifts, we shall assume $\Delta_{K\bar{\pi}} = \Delta_{\bar{K}\pi}$ and calculate the amplitudes and see if we can obtain the SU(3) violation effects in R_2 totally from the amplitude. Indeed we find that we can.

The quark-diagram amplitudes in the large N_c factorization approach are given by:

$$
\begin{aligned}
\mathcal{A}_{\bar{K}\pi} &= a_1(m_D^2 - m_K^2)f_\pi f_0^{DK}(m_\pi^2), \\
\mathcal{B}_{\bar{K}\pi} &= a_2(m_D^2 - m_\pi^2)f_K f_0^{D\pi}(m_K^2), \\
\mathcal{A}_{K\bar{\pi}} &= a_1(m_D^2 - m_\pi^2)f_K f_0^{D\pi}(m_K^2), \\
\mathcal{B}_{K\bar{\pi}} &= a_2(m_D^2 - m_\pi^2)f_K f_0^{D\pi}(m_K^2), \\
\mathcal{C}_{K\bar{\pi}} &= \mathcal{C}_{\bar{K}\pi}.
\end{aligned}
\tag{25}
$$

It is clear from Eq.(25) that $(\mathcal{B})_{K\bar{\pi}} = (\mathcal{B})_{\bar{K}\pi}$. Therefore, the SU(3)-breaking effect in R_2 comes solely from the external W-emission amplitude $\mathcal{A}$, which is $\mathcal{A}_{K\bar{\pi}}/\mathcal{A}_{\bar{K}\pi} = 1.487$, calculated from Eq.(25). The current model calculation is incapable of estimating the W-exchange amplitude $\mathcal{C}$ since the relevant form factors have to be evaluated at $q^2 = m_D^2$. Using the model calculation of the amplitude $\mathcal{A}_{\bar{K}\pi}$ given in Eq.(25) we can determine $\mathcal{C}$ from the measured branching ratio of $D^0 \to K^-\pi^+$ and $\tau(D^0) = 4.20 \times 10^{-13}s$ [6]

$$\left(\frac{\mathcal{C}}{\mathcal{A}}\right)_{\bar{K}\pi} \cong -0.13. \tag{26}$$

This shows that the W-exchange contribution is small but not negligible. Putting everything together into Eq.(2), we find

$$R_2 = 2.3. \tag{27}$$

This is in agreement with the experimental result Eq.(6). (We would like to comment that the final value of R_2 is not very sensitive to the value of Eq.(26). For example if we were to use twice of that value, we would obtain $R_2 = 2.6$.)

To conclude, we have shown that the unexpected large decay rates of the recently measured quark-mixing singly-suppressed decay $D^+ \to \pi^+\pi^0$ and the doubly-suppressed decay $D^0 \to K^+\pi^-$ by CLEO, hence the large ratios of R_1 and R_2, can be accounted for in the large-N_c factorization approach. Accumulations of small SU(3)-symmetry breaking effects in the decay constants, form factors, and mass-difference ratio lead to large values of R_1 and R_2, deviating from the SU(3)-symmetry values of unity for both. An important ingredient for our analysis to obtain the correct values of R_1 and R_2 is the relative magnitude of the form factors $f_+^{D\pi}(0) > f_+^{DK}(0)$, consistant with the recent theoretical calculation [13] and a recent CLEO measurement of $D \to \pi\ell\bar{\nu}$ [12].

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